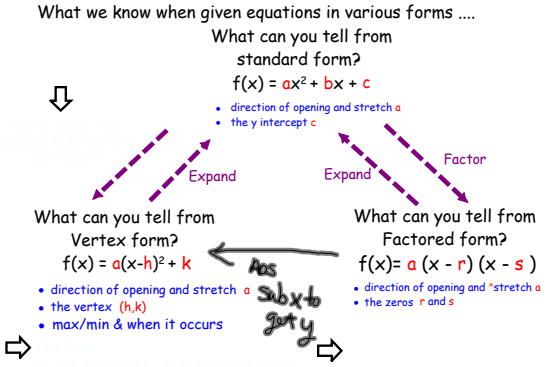
2.9 Completing the Square

What we know when given equations in various forms ....



Algebraically...let's look for a pattern...

1. $(x + 1)^2 =$ $(x + 1)(x + 1)$ $= x^2 + 2x + 1$
2. $(x - 1)^2 =$ _____ $= x^2 - 2x + 1$
3. _____ $=$ $(x + 2)(x + 2) = x^2 + 4x + 4$
4. $(x - 2)^2 =$ $(x - 2)(x - 2)$ $=$ _____
5. $(x +$__$)^2 =$ _____ $= x^2 + 6x +$ ___
6. $(x -$__$)^2 =$ _____ $= x^2 - 6x +$ ___
7. $(x -$__$)^2 =$ _____ $= x^2 - 8x +$ ___

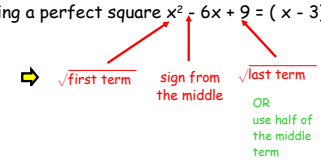
So...to find the last term ....

divide the middle coefficient by 2 and square it

Let's try...

1. $x^2 + 10x +$ ___$= ($ $)^2$
2. $x^2 + 20x +$ ___$= ($ $)^2$
3. $y^2 - 12y +$ ___$= ($ $)^2$
4. $m^2 - 3m +$ ___$= ($ $)^2$

So factoring a perfect square $x^2 - 6x + 9 = (x - 3)^2$



Complete the square

a) $x^2 - 10x + 1$ $\frac{-10}{2} = -5$

$= x^2 - 10x + 25 - 25 + 1$ $(-5)^2 = 25$

$= (x - 5)^2 - 24$

Vertex $(5, -24)$

b) $x^2 + 6x - 16$ $\frac{6}{2} = 3$ $(3)^2 = 9$

$= x^2 + 6x + 9 - 9 - 16$

$= (x + 3)^2 - 25$

Vertex $(-3, -25)$

c) $20x + x^2 + 3$

$= x^2 + 20x + 3$

$= x^2 + 20x + 100 - 100 + 3$

$= (x + 10)^2 - 97$

Vertex $(-10, -97)$

What happens when there is a number in front of x^2

Complete the square

a) $5x^2 + 30x - 16$

$= 5(x^2+6x)-16$

$= 5(x^2+6x+9-9)-16$

$= 5[(x+3)^2-9]-16$

$= 5(x+3)^2-45-16$

$\therefore 5(x+3)^2-61$

b) $-4.9x^2 + 29.4x + 7$

$-4.9(x^2-6x)+7$

ANS:

$-4.9(x-3)^2+51.1$

c) $3x^2 + 12x$

$=3(x^2+4x)$

$=3(x^2+4x+4-4)$

$=3[(x+2)^2-4]$

$=3(x+2)^2-12$

Would it be easier to find the vertex another way here?

Slightly more difficult...

Complete the square

a) $\frac{1}{4}x^2 + 5x + 2$

$=\frac{1}{4}(x^2+20x)+2$

$=\frac{1}{4}(x^2+20x+100-100)+2$

$=\frac{1}{4}[(x+10)^2-100]+2$

$=\frac{1}{4}(x+10)^2-25+2$

$=\frac{1}{4}(x+10)^2-23$

b) $-3x^2 + 5x + 1$

ANS $= -3\left(x-\frac{5}{6}\right)^2+\frac{37}{12}$

Hmwk:
Rest of Handout 2.9
& p 214 # 6 bce, 7b-f, 8, 9,13,17